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In the minimal $SU(3)_L \times U(1)_N$ gauge model with a global $L_e - L_\mu - L_\tau$ ($= L'$) symmetry and a discrete Z_4 symmetry, it is found that the interplay between neutrinos and charged leptons contained in triplets of $\psi^i = (\nu_L^i, \ell_L^i, \ell_L^{ci})$ ($i=1,2,3$) naturally leads to the large mixing angle (LMA) MSW solution. The model includes two (anti)sextet Higgs scalars, $S^{(0)}$ with $L'=0$ and $S^{(+)}$ with $L'=2$, which, respectively, couple to $\psi^1\psi^{2,3}$ for the electron mass and masses of atmospheric neutrinos and to $\psi^{2,3}\psi^{2,3}$ for the μ - and τ -masses and one-loop radiative neutrino masses relevant to solar neutrinos. This mechanism is realized by utilizing an additional residual discrete symmetry supplied by explicitly broken L' , which guarantees the absence of tree-level neutrino mass terms of the $\psi^{2,3}\psi^{2,3}$ -type. Pure rotation effects due to the diagonalization of neutrino and charged-lepton mass matrices are estimated to yield $\Delta m_\odot^2/\Delta m_{atm}^2 \lesssim (m_e/m_\mu)^{3/2} = \mathcal{O}(10^{-4})$ but the radiative effects supersede the rotation effects to yield $\Delta m_\odot^2/\Delta m_{atm}^2 = \mathcal{O}(10^{-2})$ as the LMA solution.

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I. INTRODUCTION

In the recent report on the detailed analysis of solar neutrino oscillations [1–3] done by the Super-Kamiokande collaboration [4], it has been suggested that solutions with large mixing angles are favored while solutions with small mixing angles are disfavored at the 95% confidence level. Neutrino oscillations controlled by such large mixing angle have also been observed and confirmed for atmospheric neutrinos [5]. Both observed oscillations are characterized by the same property of neutrino mixings, namely, large neutrino mixings, which are theoretically consistent with bimaximal mixing scheme [6,7]. The difference arises in their oscillation scales denoted by Δm_{atm}^2 for atmospheric neutrinos and Δm_\odot^2 for solar neutrinos, which are specified by $\Delta m_{atm}^2 \sim 3 \times 10^{-3} \text{ eV}^2$ and $\Delta m_\odot^2 \lesssim 10^{-4} \text{ eV}^2$, thereby, indicating a hierarchy of $\Delta m_{atm}^2 \gg \Delta m_\odot^2$.

Neutrino oscillations arise for massive neutrinos [8]. Since neutrino masses of $\sim 5.5 \times 10^{-2} \text{ eV}$ are implied by Δm_{atm}^2 , models for neutrino oscillations must be equipped with mechanisms generating such tiny masses of neutrinos [9–11], which include breaking of the lepton number conservation for Majorana neutrino masses. It has been long known [12] that such lepton number breaking interactions are inevitably contained in the minimal $SU(3)_L \times U(1)_N$ model for electroweak interactions (called the 331 model) [13] because a charged lepton (ℓ) and its antiparticle (ℓ^c) together with a neutrino (ν) are placed in a triplet of $SU(3)_L$ as $\psi = (\nu_L, \ell_L, \ell_L^c)^T$. As a result, the model provides a charged-lepton mass term included in $\psi\psi$, which simultaneously contains $\nu_L\nu_L$ as a neutrino mass term. Namely, the need for Majorana mass term is linked to the existence of the massive charged leptons. Therefore, the 331 model is suitable for incorporating Majorana neutrino masses [12,14–16].

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With this nice feature of the 331 model in mind, we introduce the bimaximal mixing scheme into the model to accommodate observed neutrino oscillations. One of the possible scenario for bimaximal mixing scheme is to employ a new $U(1)_{L'}$ symmetry based on $L_e - L_\mu - L_\tau (\equiv L')$ [17,18]. Neutrino mixings are predicted to be maximal for solar neutrinos with $\Delta m_\odot^2=0$ but arbitrary for atmospheric neutrinos with $\Delta m_{atm}^2 \neq 0$. The hierarchy of $\Delta m_{atm}^2 \gg \Delta m_\odot^2$ is trivially satisfied since $\Delta m_\odot^2=0$. One has to next explain how a nonvanishing Δm_\odot^2 is induced while preserving $\Delta m_{atm}^2 \gg \Delta m_\odot^2$. An elaborated mechanism is to generate Δm_\odot^2 as radiative effects [19], whose generic smallness explains the relative smallness of Δm_\odot^2 over Δm_{atm}^2 [20,21].

In this article, we construct our interactions that induce phenomenologically consistent neutrino and charged-lepton mass matrix, M_ν and M_ℓ , with appropriate radiative effects for Δm_\odot^2 . Since the model describes both matrices in a unified form, masses for charged leptons and neutrinos are correlated to each other. The mass term of $\psi\psi$ transforms as 3^* and 6, which requires 3 and/or 6^* as Higgs scalars. The coupling of the triplet Higgs scalar to $\psi\psi$ is antisymmetrized with respect to the $SU(3)_L$ -index. The spin and statistics, then, requires antisymmetrized flavor indices carried by ψ , in turn, generating antisymmetrized M_ℓ with eigenvalues corresponding to $m_\mu = m_\tau$, which contradicts with the observed hierarchy of $m_\mu \ll m_\tau$. The triplet Higgs scalar alone is not consistent with the observed lepton mass spectrum. Therefore, the sextet scalar, which provides symmetrized mass terms, is the key ingredient of the 331 model [22].¹

Because the sextet Higgs scalar contains both an $SU(2)_L$ -doublet scalar for charged leptons and an $SU(2)_L$ -triplet scalar for neutrinos, we naturally expect “degeneracy” in the structure of their mass matrices. The charged leptons should contain a diagonal mass term such as the τ -mass, which forces M_ν to have a diagonal term. However, M_ν should not contain diagonal terms to be consistent with bimaximal mixing scheme based on L' . To avoid this “degeneracy”, we use the conservation of the L' quantum number supplemented by an appropriate Z_4 parity as well as a specific breaking pattern of L' yielding a residual discrete symmetry. The collaboration of these symmetries allows neutrinos to develop no diagonal mass terms. As a result, atmospheric neutrino oscillations are controlled by those tree-level off-diagonal masses while solar neutrino oscillations are to be induced by one-loop effects.²

In the following section, we first introduce particles and their interactions and discuss how the required vacuum alignment is realized. Section III deals with the diagonalization of M_ν , which reflects rotation effects due to the diagonalization of M_ℓ . The bimaximal condition on the neutrino masses determines a pattern of the matrix elements for the electron mass. One-loop radiative mass of ν_e is also calculated. The discussions in Sec.IV include to examine Δm_\odot^2 by utilizing the correlation between M_ν and M_ℓ and to confirm that the partial “degeneracy” between M_ν and M_ℓ suppresses the rotation effects to yield the LMA solution based on the radiative effects. The final section is devoted to summary and discussions.

II. MODEL

The model contains neutrinos and charged leptons in triplets of $\psi_\alpha^i = (\nu_\alpha^i, \ell_\alpha^i, \ell_\alpha^{ci})^T$ ($i, \alpha=1,2,3$)³ with $L' = 1$ for ψ^1 and $L' = -1$ for $\psi^{2,3}$, where the superscript c of ℓ_α^{ci} stands for the charge conjugation, and two (anti-)sextet Higgs scalars $S^{(0,+)\alpha\beta}$ with $L'=(0, 2)$ for $(S^{(0)}, S^{(+)})$. Since $S^{(0)}$ can couple to $\psi^1\psi^i$ ($i=2,3$), $\nu_{eL} - \nu_L^i$ mixing terms are generated by $\langle 0|S^{(0)11}|0\rangle \neq 0$ and are to be responsible for the bimaximal structure. On the other hand, m_τ (and also m_μ) are generated by $\langle 0|S^{(+23)}|0\rangle \neq 0$ via its coupling to $\psi^i\psi^j$ ($i, j=2,3$). Since $\psi^i\psi^j$ also supply couplings to $\nu_{\mu L}$ and $\nu_{\tau L}$, if $\langle 0|S^{(+11)}|0\rangle \neq 0$ is also induced, then it disturbs the nice feature of the bimaximal mixing scheme based on L' . Therefore, in order for the present approach to be acceptable, the simultaneous alignment of $\langle 0|S^{(+11)}|0\rangle = 0$ and $\langle 0|S^{(+23)}|0\rangle \neq 0$ should be dynamically allowed. It is demonstrated that this vacuum alignment of $S^{(+11)}$ and $S^{(+23)}$ is indeed possible to occur as a result of the emergence of a residual discrete symmetry specific to $U(1)_{L'}$.

A. Particles

The 331 model is specified by the $U(1)_N$ -charge. Let $N/2$ be the $U(1)_N$ quantum number, then the hypercharge, Y , is given by $Y = -\sqrt{3}\lambda^8 + N$ and the electric charge Q_{em} is given by $Q_{em} = (\lambda^3 + Y)/2$, where λ^a is the $SU(3)$

¹The use of an $SU(3)_L$ -singlet vector-like heavy “electron” without introducing a sextet scalar [23] has been advocated to describe consistent mass spectra of neutrinos and charged leptons [16].

²Similar discussions based on L' and a discrete symmetry have been done in Ref. [24].

³Throughout this paper, the roman letters of i, j, \dots and the Greek letters of α, β, \dots , respectively, stand for the three families and three $SU(3)_L$ -indices.

generator with $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$ ($a, b = 1 \dots 8$). Let us first summarize the whole particle content in our model, where the quantum numbers are specified by $(SU(3)_L, U(1)_N; U(1)_{L'})$:

$$-\psi^1 = (\nu_L^1, \ell_L^1, \ell_L^{c1})^T : (\mathbf{3}, 0; 1), \quad \psi^{i=2,3} = (\nu_L^i, \ell_L^i, \ell_L^{ci})^T : (\mathbf{3}, 0; -1), \quad (1)$$

for leptons,

$$Q_L^{i=1,2} = (d_L^i, -u_L^i, J_L^i)^T : (\mathbf{3}^*, -1/3; 0), \quad Q_L^3 = (u_L^3, d_L^3, J_L^3)^T : (\mathbf{3}, 2/3; 0), \\ u_R^{1,2,3} : (\mathbf{1}, 2/3; 0), \quad d_R^{1,2,3} : (\mathbf{1}, -1/3; 0), \quad J_R^{1,2} : (\mathbf{1}, -4/3; 0), \quad J_R^3 : (\mathbf{1}, 5/3; 0), \quad (2)$$

for quarks

$$\eta = (\eta^0, \eta^-, \eta^+)^T : (\mathbf{3}, 0; 0), \quad \rho = (\rho^+, \rho^0, \rho^{++})^T : (\mathbf{3}, 1; 0), \quad \chi = (\chi^-, \chi^{--}, \chi^0)^T : (\mathbf{3}, -1; 0), \quad (3)$$

for triplet Higgs scalars, and

$$S^{(0)} = \begin{pmatrix} s_\nu^{(0)} & s^{(0)+} & s^{(0)-} \\ s^{(0)+} & s^{(0)++} & s_\ell^{(0)} \\ s^{(0)-} & s_\ell^{(0)} & s^{(0)--} \end{pmatrix} : (\mathbf{6}^*, 0; 0), \quad S^{(+)} = \begin{pmatrix} s_\nu^{(+)} & s^{(++)} & s^{(+)-} \\ s^{(++)} & s^{(++)} & s_\ell^{(+)} \\ s^{(+)-} & s_\ell^{(+)} & s^{(++)} \end{pmatrix} : (\mathbf{6}^*, 0; 2), \quad (4)$$

for (anti-)sextet Higgs scalars. These Higgs scalars have the following vacuum expectation values (VEV's):

$$\langle 0|\eta|0\rangle = (v_\eta, 0, 0)^T, \quad \langle 0|\rho|0\rangle = (0, v_\rho, 0)^T, \quad \langle 0|\chi|0\rangle = (0, 0, v_\chi)^T, \quad (5)$$

$$\langle 0|S^{(0)}|0\rangle = \begin{pmatrix} v_\nu^{(0)} & 0 & 0 \\ 0 & 0 & v_\ell^{(0)} \\ 0 & v_\ell^{(0)} & 0 \end{pmatrix}, \quad \langle 0|S^{(+)}|0\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & v_\ell^{(+)} \\ 0 & v_\ell^{(+)} & 0 \end{pmatrix}, \quad (6)$$

and quarks and leptons will acquire masses via these VEV's, where the orthogonal choice of the VEV's in Eq.(5) and the vanishing VEV of $\langle 0|S^{(+)}|0\rangle$ in Eq.(6) are to be ensured by appropriate Higgs interactions.

The pure $SU(3)_L$ -anomaly is cancelled in a vectorial manner. The anomalies from triplets of the three families of leptons and of the three colors of the third family of quarks are cancelled by those from antitriplets of three colors of the first and second families of quarks [13]. Other anomalies including $U(1)_N$ are also cancelled. The scale of the 331 model is set by v_χ , yielding $SU(3)_L \times U(1)_N \rightarrow SU(2)_L \times U(1)_Y$ and we expect the magnitude of v_χ is of order TeV.

B. Interactions

To realize phenomenologically viable interactions, we further impose a Z_4 parity on the model, where $Z_4=+$ for ψ^1, η and $S^{(+)}$; $-$ for $\psi^{2,3}$ and $S^{(0)}$; i for ρ and χ and similarly for quarks. Hereafter, we restrict ourselves to the lepton sector. The Yukawa interactions for leptons are, then, controlled by the following lagrangian:

$$-\mathcal{L}_Y = \sum_{i=2,3} f_{1i} \overline{(\psi^1)^c} \psi^i S^{(0)} + \sum_{i,j=2,3} \frac{1}{2} f_{ij} \overline{(\psi^i)^c} \psi^j S^{(+)} + (\text{h.c.}), \quad (7)$$

where f 's denote the Yukawa couplings with $f_{ij} = f_{ji}$. The possible term of the $\psi\psi\eta$ -type is forbidden by Z_4 . The Higgs interactions are described by self-Hermitian terms composed of $\phi_\alpha \phi^{c\beta}$ ($\phi=\eta, \rho, \chi$) and $S^{(0,+)\alpha\beta} S_{\alpha'\beta'}^{(0,+)^c}$, which include the potential term of $V_{\eta\rho\chi}$:

$$V_{\eta\rho\chi} = \lambda_{\eta\rho} |\eta \times \rho|^2 + \lambda_{\rho\chi} |\rho \times \chi|^2 + \lambda_{\chi\eta} |\chi \times \eta|^2, \quad (8)$$

where λ 's represent coupling constants and $(a \times b)^\alpha \equiv \epsilon^{\alpha\beta\gamma} a_\beta b_\gamma$ and by the non-self-Hermitian terms in

$$V_0 = \mu_0 \rho S^{(0)} \chi + \lambda_1 \eta \eta S^{(0)c} S^{(0)c} + \lambda_2 \eta S^{(0)} \rho^c \chi^c + (\text{h.c.}), \quad (9)$$

where μ_0 denotes a mass scale and $\lambda_{1,2}$ stand for coupling constants. There is a leptonic number associated with ψ , which can be taken to be -2 for $S^{(0,+)}$ and 0 for all others. Since the L_ℓ conservation is broken both spontaneously by $S^{(0,+)}$ and explicitly by V_0 , there is no harmful Nambu-Goldstone boson.

Although the lepton number cannot be globally assigned to $SU(3)_L$ -multiplets, to argue Majorana masses for neutrinos, it is useful to define the lepton number L to classify interactions [25] in the similar way used for the electric charge defined in the standard $SU(2)_L \times U(1)_Y$ model. Following the classification lately done in Ref. [26], the L -number can be introduced in the form of

$$L = 4\lambda^8/2\sqrt{3} + \mathcal{L}, \quad (10)$$

where $\mathcal{L} = 2$ for $J_R^{1,2}$; $= 4/3$ for χ ; $= 2/3$ for $Q_L^{1,2}$; $= 1/3$ for $\psi^{1,2,3}$; $= 0$ for $u_R^{1,2,3}$ and $d_R^{1,2,3}$; $= -2/3$ for Q_L^3 , η , ρ and $S^{(0,+)}$; $= -2$ for J_R^3 , which yields $L = (1, 1, -1)$ for $\psi^{1,2,3}$ as usual. The assignment for leptons and Higgs scalars is shown in TABLE I. Among our interactions constrained by L' , only $\eta S^{(0)} \rho^c \chi^c$ breaks the L conservation; therefore, it is this interaction that gives the nonvanishing $v_\nu^{(0)}$. A few comments are in order to support our choice of VEV's of $v_{\eta,\rho,\chi}$ and $v_{\nu,\ell}^{(0)}$:

1. The nonvanishing VEV of $\langle 0|S^{(0)11}|0\rangle$ is triggered by the L -violating $\eta S^{(0)} \rho^c \chi^c$ for $\lambda_2 < 0$ since $\langle 0|\eta_1|0\rangle \neq 0$, $\langle 0|\rho_2|0\rangle \neq 0$ and $\langle 0|\chi_3|0\rangle \neq 0$;
2. The nonvanishing VEV of $\langle 0|S^{(0)23}|0\rangle$ is triggered by the L -conserving $\rho S^{(0)} \chi$ for $\mu_0 < 0$ since $\langle 0|\rho_2|0\rangle \neq 0$ and $\langle 0|\chi_3|0\rangle \neq 0$ and by the L -conserving $\eta \eta S^{(0)c} S^{(0)c}$ for $\lambda_1 < 0$ since $\langle 0|\eta_1|0\rangle \neq 0$;
3. The orthogonal choice of VEV's of η , ρ and χ as in Eq.(5) is supported by $V_{\eta\rho\chi}$ if all λ 's are negative. It is because $V_{\eta\rho\chi}$ gets lowered if η , ρ and χ develop VEV's. So, one can choose VEV's such that $\langle 0|\eta_1|0\rangle \neq 0$, $\langle 0|\rho_2|0\rangle \neq 0$ and $\langle 0|\chi_3|0\rangle \neq 0$.

Thus, these VEV's are dynamically aligned by our Higgs potential. On the other hand, we cannot argue whether $\langle 0|S^{(+11,23)}|0\rangle$ acquire VEV's or not since there are no triggers such as an L -violating $\eta S^{(+)} \eta$ and an L -conserving $\rho S^{(+)} \chi$, which can, respectively, align $\langle 0|S^{(+11)}|0\rangle$ and $\langle 0|S^{(+23)}|0\rangle$ to be nonvanishing. However, so far these interactions are forbidden by the L' conservation. To invoke such dynamical alignment, thus, requires L' -breaking interactions.

To introduce L' -breaking effects calls for careful consideration because of the specific alignment of VEV's of $S^{(+)}$, *i.e.* $\langle 0|S^{(+11)}|0\rangle = 0$ and $\langle 0|S^{(+23)}|0\rangle \neq 0$. Its breaking readily induces dangerous interactions that creates $\langle 0|S^{(+11)}|0\rangle$ such as tadpole interactions for $S^{(+11)}$. Owing to the vacuum alignment of $\langle 0|S^{(+11)}|0\rangle = 0$, L' -breaking interactions should conserve L . There is one candidate that can keep this specific alignment, which is an L -conserving $\eta \eta S^{(+c)} S^{(+c)}$:

$$V_b = \lambda_b \eta \eta S^{(+c)} S^{(+c)} + (\text{h.c.}), \quad (11)$$

where λ_b represents a breaking of the L' conservation. Since $U(1)_{L'}$ is broken by the unit of $|Q_{L'}|=4$, there is still a residual conservation due to a Z_4 symmetry of $\exp(i\pi Q_{L'}/2)$ to be denoted by $Z_{L'}$, which is shown in TABLE I together with L' and Z_4 . This $Z_{L'}$ symmetry can be used to constrain possible forms of L' -breaking interactions. The mass terms for μ and τ are created by $\langle 0|S^{(+23)}|0\rangle \neq 0$, which is now supported by V_b for $\lambda_b < 0$. Therefore, V_b serves as a trigger for $\langle 0|S^{(+23)}|0\rangle \neq 0$.

The VEV of $\langle 0|S^{(+11)}|0\rangle$ would be created by the following L -violating interactions of

- $\text{Tr}(S^{(0)\dagger} S^{(+)})$ since $\langle 0|S^{(0)11}|0\rangle \neq 0$,
- $\eta S^{(+)} \eta$ since $\langle 0|\eta_1|0\rangle \neq 0$,
- $\eta S^{(+)} \rho^c \chi^c$ since $\langle 0|\eta_1|0\rangle \neq 0$, $\langle 0|\rho_2|0\rangle \neq 0$ and $\langle 0|\chi_3|0\rangle \neq 0$,
- $\rho \chi S^{(0)c} S^{(+c)}$ and $\rho \chi S^{(+c)} S^{(+c)}$ since $\langle 0|\rho_2|0\rangle \neq 0$, $\langle 0|\chi_3|0\rangle \neq 0$ and $\langle 0|S^{(0,+23)}|0\rangle \neq 0$ and
- $\text{Tr}(S^{(0)} S^{(0)} S^{(+)})$, $\text{Tr}(S^{(0)} S^{(+)} S^{(+)})$ and $\text{Tr}(S^{(+)} S^{(+)} S^{(+)})$ since $\langle 0|S^{(0,+23)}|0\rangle \neq 0$.

Fortunately, all of these interactions are forbidden by the interplay between Z_4 and $Z_{L'}$. Tabulated in TABLE II are Z_4 and $Z_{L'}$ for non-self-Hermitian Higgs interactions allowed by $SU(3)_L \times U(1)_N$, where the reader can recognize how these dangerous interactions are excluded. At this moment, we reach the plausible situation, where the alignment of

$$\langle 0|S^{(+11)}|0\rangle = 0, \quad \langle 0|S^{(+23)}|0\rangle \neq 0 \quad (12)$$

is dynamically supported.

The resulting mass matrices for charged leptons and neutrinos take the form of

$$M_\ell = \begin{pmatrix} 0 & \delta m_{12}^\ell & \delta m_{13}^\ell \\ \delta m_{12}^\ell & m_{22}^\ell & m_{23}^\ell \\ \delta m_{13}^\ell & m_{23}^\ell & m_{33}^\ell \end{pmatrix}, \quad M_\nu = \begin{pmatrix} \delta m_{11}^{rad} & \delta m_{12}^\nu & \delta m_{13}^\nu \\ \delta m_{12}^\nu & \delta m_{22}^{rad} & \delta m_{23}^{rad} \\ \delta m_{13}^\nu & \delta m_{23}^{rad} & \delta m_{33}^{rad} \end{pmatrix}, \quad (13)$$

where $m_{ij}^{\ell,\nu}$ are expressed in terms of VEV's:

$$\delta m_{1i}^\ell = f_{1i} v_\ell^{(0)}, \quad \delta m_{1i}^\nu = f_{1i} v_\nu^{(0)}, \quad m_{ij}^\ell = f_{ij} v_\ell^{(+)}, \quad (14)$$

leading to the proportionality of

$$\delta m_{1i}^\nu \propto \delta m_{1i}^\ell, \quad (15)$$

and δm_{ij}^{rad} stand for one-loop radiative masses to be estimated in the next subsection. We simply assume $\delta m_{1i}^\ell \ll m_{ij}^\ell$ ($i, j=2,3$) so as to reproduce the hierarchy of $m_e \ll m_{\mu,\tau}$ by the mechanism discussed in Ref. [27].

Since the magnitude of neutrino masses δm_{1i}^ν is controlled by $v_\nu^{(0)}$, let us explain how naturally small $v_\nu^{(0)}$ is obtained. An estimate of

$$v_\nu^{(0)} \sim -\lambda_2 v_\eta v_\rho / v_\chi \quad (16)$$

can be found in the analyses of Ref. [15] applied to our Higgs potential. It calls for $\lambda_2 < 0$ as has been expected in the vacuum alignment. The smallness of $v_\nu^{(0)}$ is, then, ascribed to either $|\lambda_2| \ll 1$ [26] or $v_\chi \gg v_{\eta,\rho}$ [28], the latter of which is known as the type II seesaw mechanism [29], where we use the mechanism due to $|\lambda_2| \ll 1$ by simply expecting that v_χ is of order TeV. One will find that $(-\lambda_2) \sim \text{a few} \times 10^{-7}$ provides neutrino masses about 5.5×10^{-2} eV. This is in fact a fine-tuning of λ_2 to reproduce neutrino masses so as to meet the value of 5.5×10^{-2} . However, we have already known this tiny magnitude, which is give by m_e/m_t . The relative smallness of m_e is linked to the underlying chiral symmetry recovered at the limit of $m_e \rightarrow 0$. This feature dictates the so-called ‘‘naturalness’’ [30]. Similarly, since the L conservation is respected in the limit of $\lambda_2 \rightarrow 0$, the process of taking $|\lambda_2| \ll 1$ is also regarded to be natural although both cases require other new physics to explain why such tiny values are realized.

Our program to discuss patterns of neutrino oscillations is based on the use of the proportionality of δm_{1i}^ν to δm_{1i}^ℓ indicated by Eq.(15). Since to generate the electron mass and to reproduce Δm_{atm}^2 will, respectively, determine δm_{1i}^ℓ and δm_{1i}^ν , which yield the estimation of $v_\nu^{(0)}/v_\ell^{(0)}$. The magnitude of $v_\nu^{(0)}$ derived in Eq.(16) now determines that of λ_2 once appropriate values of VEV's are given and, then, $\delta m_{11,i,j}^{rad}$ ($i, j=2,3$) to be defined in Eqs.(23) and (24) are calculable for this estimated λ_2 . In the end, solar neutrino oscillations controlled by $\delta m_{11,i,j}^{rad}$ are found to be consistent with the LMA solution.

A. Charged Lepton Masses

We first discuss the diagonalization of M_ℓ . The following unitary matrix U_ℓ is obtained under the approximation of $\delta m_{1i}^\ell \ll m_{ij}^\ell$ ($i, j=2,3$) and transforms M_ℓ into $M_\ell^{diag} = U_\ell^\dagger M_\ell U_\ell = \text{diag.}(-m_e, m_\mu, m_\tau)$:

$$U_\ell = \begin{pmatrix} 1 - (\delta U_\mu^2 + \delta U_\tau^2)/2, & \delta U_\mu & \delta U_\tau \\ -(c_\alpha \delta U_\mu + s_\alpha \delta U_\tau) & c_\alpha (1 - \delta U_\mu^2/2) - s_\alpha \delta^2 V_{\tau\mu} & s_\alpha (1 - \delta U_\tau^2/2) + c_\alpha \delta^2 V_{\mu\tau} \\ s_\alpha \delta U_\mu - c_\alpha \delta U_\tau & -s_\alpha (1 - \delta U_\mu^2/2) - c_\alpha \delta^2 V_{\tau\mu} & c_\alpha (1 - \delta U_\tau^2/2) - s_\alpha \delta^2 V_{\mu\tau} \end{pmatrix}, \quad (17)$$

up to the second order of $\delta U_{\mu,\tau}$ with

$$\begin{aligned} \delta U_\mu &= ((c_\alpha^2 - s_\alpha^2) \delta m_{12}^\ell - 2c_\alpha s_\alpha \delta m_{13}^\ell) / \lambda_-, \quad \delta U_\tau = (2c_\alpha s_\alpha \delta m_{12}^\ell + (c_\alpha^2 - s_\alpha^2) \delta m_{13}^\ell) / \lambda_+, \\ \delta^2 V_{ij} &= \lambda_i \delta U_i \delta U_j / (\lambda_+ - \lambda_-), \quad \lambda_\pm = \left(m_{22}^\ell + m_{33}^\ell \pm \sqrt{(m_{22}^\ell - m_{33}^\ell)^2 + 4m_{23}^{\ell 2}} \right) / 2, \\ c_\alpha &\equiv \cos \alpha = \sqrt{(m_{33}^\ell - \lambda_-) / (\lambda_+ - \lambda_-)}, \quad s_\alpha \equiv \sin \alpha = \sqrt{(\lambda_+ - m_{33}^\ell) / (\lambda_+ - \lambda_-)}, \end{aligned} \quad (18)$$

where $i, j = \mu, \tau$ for $\delta^2 V_{ij}$, $\lambda_\mu = \lambda_-$ and $\lambda_\tau = \lambda_+$. The masses of e , μ and τ are given by

$$m_e = \Delta m_{12} + \Delta m_{13}, \quad m_\mu = \lambda_- + \Delta m_{12}, \quad m_\tau = \lambda_+ + \Delta m_{13}, \quad (19)$$

where

$$\Delta m_{12} = ((c_\alpha^2 - s_\alpha^2) \delta m_{12}^\ell - 2c_\alpha s_\alpha \delta m_{13}^\ell)^2 / \lambda_-, \quad \Delta m_{13} = (2c_\alpha s_\alpha \delta m_{12}^\ell + (c_\alpha^2 - s_\alpha^2) \delta m_{13}^\ell)^2 / \lambda_+. \quad (20)$$

To be consistent with the observed mass pattern of the charged lepton mass matrix, we should present a mechanism generating plausible mass textures such as the hierarchical pattern [27] and the democratic pattern [31]. However, in the present context of the model, we are not aiming at clarifying all the physical aspect of the lepton sector but, instead, at clarifying the correlation of neutrino mass matrix with charged lepton mass matrix. Here, we simply parameterize m_{ij}^ℓ by adopting two distinct ways to realize $m_\mu \ll m_\tau$: hierarchical and democratic mass textures and discuss their influences in neutrino mixings. The resulting parameterization is given by

1. the hierarchical pattern of $m_{22,23}^\ell \ll m_{33}^\ell$ and

$$m_e \approx \delta m_{12}^{\ell^2} / m_\mu + \delta m_{13}^{\ell^2} / m_\tau, \quad m_\mu \approx m_{22}^\ell + (m_{23}^{\ell^2} / m_\tau), \quad m_\tau \approx m_{33}^\ell - (m_{23}^{\ell^2} / m_\tau), \\ \delta U_\mu \approx \delta m_{12}^\ell / m_\mu, \quad \delta U_\tau \approx \delta m_{13}^\ell / m_\tau, \quad (21)$$

leading to $\cos \alpha \approx 1$ with $\sin \alpha \approx m_{23}^\ell / m_{33}^\ell$ ($\ll 1$) and

2. the democratic pattern of $m_{22}^\ell \approx m_{23}^\ell \approx m_{33}^\ell$ and

$$m_e \approx \delta m_{13}^{\ell^2} / m_\mu + \delta m_{12}^{\ell^2} / m_\tau, \quad m_\mu \approx \Delta m, \quad m_\tau \approx 2m_0 - \Delta m, \\ \delta U_\mu \approx -\delta m_{13}^\ell / m_\mu, \quad \delta U_\tau \approx \delta m_{12}^\ell / m_\tau, \quad (22)$$

where m_0 and Δm specify $m_{22}^\ell = m_{33}^\ell = m_0$ and $m_{23}^\ell = m_0 - \Delta m$ ($m_0 \gg \Delta m$), leading to $\cos \alpha = \sin \alpha = 1/\sqrt{2}$.

Therefore, we have found the estimate of $\delta m_{12,13}^\ell$ that reproduces m_e .

B. Radiative Masses

Neutrinos acquire radiative masses of $\delta m_{11,ij}^{rad}$ ($i, j=2,3$) generated by one-loop interactions corresponding to diagrams depicted in FIG.1 and FIG.2. It is convenient to shift the basis for charged-leptons from the original one for M_ℓ to the intermediate one, where μ and τ have diagonal masses of λ_- and λ_+ , respectively. The corrections due to $\delta m_{12,13}^\ell$ are neglected because of $\delta m_{12,13}^\ell \ll m_{\mu,\tau}$. In this basis, the couplings of $S^{(0)}$ to the lepton triplets are described by $F_{1i} \psi^1 \Psi^i$ ($i=2,3$) with $(\Psi^2, \Psi^3) = (c_\alpha \psi^2 - s_\alpha \psi^3, s_\alpha \psi^2 + c_\alpha \psi^3)$ and F_{1i} are the rotated f_{1i} -couplings similarly defined as $(F_{12}, F_{13}) = (c_\alpha f_{12} - s_\alpha f_{13}, s_\alpha f_{12} + c_\alpha f_{13})$, which appear in the figures.

The result of the calculation of $\delta m_{11,ij}^{rad}$ ($i, j=2,3$) is given by

$$\delta m_{11}^{rad} = 2\lambda_2 \mu_0 \sum_{i=2,3} F_{1i}^2 m_{\ell^i} \left[I(m_{\ell^i}^2, m_{s^{(0)+}}^2, m_{s^{(0)-}}^2, m_{\rho^+}^2) v_\chi^2 + I(m_{\ell^i}^2, m_{s^{(0)+}}^2, m_{s^{(0)-}}^2, m_{\chi^-}^2) v_\rho^2 \right] v_\eta \\ + 2\lambda_2 \lambda_1 v_\ell^{(0)} F_{1i}^2 m_{\ell^i} \left[I(m_{\ell^i}^2, m_{s^{(0)+}}^2, m_{s^{(0)-}}^2, m_{\eta^+}^2) + I(m_{\ell^i}^2, m_{s^{(0)-}}^2, m_{s^{(0)+}}^2, m_{\eta^-}^2) \right] v_\eta v_\rho v_\chi, \quad (23)$$

$$\delta m_{ij}^{rad} = 2\lambda_2 \lambda_b v_\ell^{(0)} F_{1i} F_{1j} m_{\ell^i} \left[I(m_{\ell^i}^2, m_{s^{(0)+}}^2, m_{s^{(0)-}}^2, m_{\eta^+}^2) + I(m_{\ell^i}^2, m_{s^{(0)-}}^2, m_{s^{(0)+}}^2, m_{\eta^-}^2) \right] v_\eta v_\rho v_\chi, \quad (24)$$

for $i, j=2,3$ and $m_e=0$ in this basis, with $m_{\ell^2} = \lambda_- \approx m_\mu$ and $m_{\ell^3} = \lambda_+ \approx m_\tau$ as in Eq.(18), where

$$I(a, b, c, d) = \frac{1}{16\pi^2} \frac{J(a, b, d) - J(a, c, d)}{b - c}, \quad (25)$$

$$J(a, b, c) = \frac{a \ln a}{(a-b)(a-c)} + \frac{b \ln b}{(b-a)(b-c)} + \frac{c \ln c}{(c-b)(c-a)}. \quad (26)$$

The Majorana mass δm_{11}^{rad} for ν_e is induced by effective couplings corresponding to FIG.1:

$$(\eta^\dagger \psi^1) \epsilon^{\alpha\beta\gamma} \psi_\alpha^1 \phi_\beta (\phi S^{(+)}_\gamma)_\gamma, \quad (27)$$

for $\phi=\rho$ or χ , and

$$\epsilon^{\alpha\beta\gamma} \epsilon^{\alpha'\beta'\gamma'} \psi_\alpha^1 \psi_{\alpha'}^1 (S^{(+)}_{\beta\beta'})^c_{\gamma\gamma'} (S^{(0)})^c_{\gamma\gamma'} (\epsilon^{\alpha\beta\gamma} \eta_\alpha \rho_\beta \chi_\gamma) \quad (28)$$

with similar terms scrambled by possible permutations of the fields. Both couplings conserve L' . On the other hand, the same effective coupling as Eq.(28) but with the replacement of $\lambda_2 \rightarrow \lambda_b$, thus of the L' -breaking type, yields δm_{ij}^{rad} ($i, j=2,3$) for $\nu_{\mu,\tau} - \nu_{\mu,\tau}$, which correspond to FIG.2.

C. Neutrino Masses

Now, let us turn to examining the neutrino sector. The neutrino mass matrix is also transformed into M_ν^{weak} by U_ℓ of Eq.(17) to maintain diagonal weak currents and is calculated, up to the second order of $\delta U_{\mu,\tau}$, to be:

$$M_\nu^{weak} = U_\ell^\dagger M_\nu U_\ell \approx \begin{pmatrix} \delta M_{11}, & M_{12} & M_{13} \\ M_{12} & \delta M_{22} & \delta M_{23} \\ M_{13} & \delta M_{23} & \delta M_{33} \end{pmatrix}, \quad (29)$$

where

$$\begin{aligned} M_{12} &= N_{12} (1 - 2\delta U_\mu^2 - \delta U_\tau^2/2) - N_{13} (\delta^2 V_{\tau\mu} + \delta U_\mu \delta U_\tau), \\ M_{13} &= N_{13} (1 - 2\delta U_\tau^2 - \delta U_\mu^2/2) + N_{12} (\delta^2 V_{\mu\tau} - \delta U_\mu \delta U_\tau), \\ \delta M_{11} &= \delta m_{11}^{rad} - 2(N_{12} \delta U_\mu + N_{13} \delta U_\tau), \\ \delta M_{22} &= 2N_{12} \delta U_\mu + c_\alpha^2 \delta m_{22}^{rad} - 2c_\alpha s_\alpha \delta m_{23}^{rad} + s_\alpha^2 \delta m_{33}^{rad}, \\ \delta M_{23} &= N_{13} \delta U_\mu + N_{12} \delta U_\tau + c_\alpha s_\alpha (\delta m_{22}^{rad} - \delta m_{33}^{rad}) + (c_\alpha^2 - s_\alpha^2) \delta m_{23}^{rad}, \\ \delta M_{33} &= 2N_{13} \delta U_\tau + s_\alpha^2 \delta m_{22}^{rad} + 2c_\alpha s_\alpha \delta m_{23}^{rad} + c_\alpha^2 \delta m_{33}^{rad} \end{aligned} \quad (30)$$

with

$$N_{12} = c_\alpha \delta m_{12}^\nu - s_\alpha \delta m_{13}^\nu, \quad N_{13} = c_\alpha \delta m_{13}^\nu + s_\alpha \delta m_{12}^\nu. \quad (31)$$

It is obvious to see that M_ν^{weak} satisfies $\text{Tr}(M_\nu^{weak}) = \text{Tr}(M_\nu) = \delta m_{11}^{rad} + \delta m_{22}^{rad} + \delta m_{33}^{rad}$ and $\text{Tr}(M_\nu^{weak T} M_\nu^{weak}) = \text{Tr}(M_\nu^T M_\nu) = 2(\delta m_{12}^\nu{}^2 + \delta m_{13}^\nu{}^2)$. Since $|M_{12,13}| \gg |\delta M_{ij}|$, the diagonal neutrino masses given by $(m_{\nu 1}, -m_{\nu 2}, m_{\nu 3})$ are calculated to be:

$$m_{\nu 1} = m_\nu + m_{\nu 1}^{(1)} + m_{\nu 1}^{(2)}, \quad m_{\nu 2} = m_\nu - m_{\nu 1}^{(1)} + m_{\nu 1}^{(2)}, \quad m_{\nu 3} = m_{\nu 3}^{(1)}, \quad (32)$$

up to the second order of δM_{ij} equivalent to the second order of $\delta U_{\mu,\tau}$, where

$$\begin{aligned} m_\nu &= \sqrt{M_{12}^2 + M_{13}^2}, \quad m_{\nu 1}^{(1)} = (\delta M_{11} + \cos^2 \vartheta_\nu \delta M_{22} + 2 \cos \vartheta_\nu \sin \vartheta_\nu \delta M_{23} + \sin^2 \vartheta_\nu \delta M_{33}) / 2, \\ m_{\nu 3}^{(1)} &= \sin^2 \vartheta_\nu \delta M_{22} - 2 \cos \vartheta_\nu \sin \vartheta_\nu \delta M_{23} + \cos^2 \vartheta_\nu \delta M_{33}, \quad m_{\nu 1}^{(2)} = \frac{M_{12}^2}{2m_\nu} + \frac{M_{13}^2}{m_\nu} \end{aligned} \quad (33)$$

with $\cos \vartheta_\nu = M_{12}/m_\nu$ and $\sin \vartheta_\nu = M_{13}/m_\nu$ as the mixing angle for atmospheric neutrinos and

$$\begin{aligned} M'_{12} &= (-\delta M_{11} + \cos^2 \vartheta_\nu \delta M_{22} + 2 \cos \vartheta_\nu \sin \vartheta_\nu \delta M_{23} + \sin^2 \vartheta_\nu \delta M_{33}) / 2, \\ M'_{13} &= ((\cos^2 \vartheta_\nu - \sin^2 \vartheta_\nu) \delta M_{23} + \cos \vartheta_\nu \sin \vartheta_\nu (\delta M_{33} - \delta M_{22})) / \sqrt{2}. \end{aligned} \quad (34)$$

The masses of $m_{\nu 1}^{(1)}$ and $M'_{12,13}$ can be further converted into:

$$m_{\nu 1}^{(1)} = \frac{\delta m_{11}^{rad} + \delta m_{22}^{rad}}{2} + \frac{(M_{13} \delta U_\mu - M_{12} \delta U_\tau) (M_{12} N_{13} - M_{13} N_{12})}{m_\nu^2}, \quad (35)$$

$$M'_{12} = N_{12} \delta U_\mu + N_{13} \delta U_\tau + \frac{(M_{12} \delta U_\mu + M_{13} \delta U_\tau) (M_{12} N_{12} + M_{13} N_{13})}{m_\nu^2}, \quad (36)$$

$$M'_{13} = \frac{1}{\sqrt{2}} \left[N_{13} \delta U_\mu - N_{12} \delta U_\tau - 2 \frac{(M_{13} \delta U_\mu - M_{12} \delta U_\tau) (M_{12} N_{12} + M_{13} N_{13})}{m_\nu^2} \right], \quad (37)$$

where other irrelevant terms involving δm_{ij}^{rad} are omitted. The mass of $m_{\nu 3}^{(1)}$ is obtained by noticing the relation of $\text{Tr}(M_\nu^{weak}) = \delta m_{11}^{rad} + \delta m_{22}^{rad} + \delta m_{33}^{rad}$ that yields

$$2m_{\nu 1}^{(1)} + m_{\nu 3}^{(1)} = \delta m_{11}^{rad} + \delta m_{22}^{rad} + \delta m_{33}^{rad}, \quad (38)$$

from which we find that

$$m_{\nu 3}^{(1)} = \delta m_{33}^{rad} - 2 \frac{(M_{13}\delta U_\mu - M_{12}\delta U_\tau)(M_{12}N_{13} - M_{13}N_{12})}{m_\nu^2}. \quad (39)$$

To enhance the bimaximal structure in M_ν^{weak} is achieved by setting $|M_{12}| \approx |M_{13}|$, where $M_{12,13}$ can be approximated to be:

$$M_{12} \approx N_{12} = c_\alpha \delta m_{12}^\nu - s_\alpha \delta m_{13}^\nu, \quad M_{13} \approx N_{13} = c_\alpha \delta m_{13}^\nu + s_\alpha \delta m_{12}^\nu, \quad (40)$$

because $|\delta U_{\mu,\tau}| \ll 1$. The condition of $|M_{12}| \approx |M_{13}|$ thus, requires, for the hierarchical case with $c_\alpha \approx 1$ and $s_\alpha \approx 0$,

$$\delta m_{12}^\nu \approx \delta m_{13}^\nu, \quad (41)$$

and, for the democratic case with $c_\alpha = s_\alpha = 1/\sqrt{2}$,

$$\delta m_{12}^\nu \gg \delta m_{13}^\nu \quad \text{or} \quad \delta m_{12}^\nu \ll \delta m_{13}^\nu. \quad (42)$$

The essence of the present research lies in the fact that the bimaximal mixing scheme for neutrinos, in turn, determines how the electron mass is derived. The bimaximal condition of Eq.(41) combined with Eq.(21) yields

$$\delta m_{12}^\ell \approx \delta m_{13}^\ell \approx \sqrt{m_e m_\mu}, \quad \delta U_\mu \approx \sqrt{m_e/m_\mu}, \quad \delta U_\tau \approx \sqrt{m_e/m_\tau}, \quad (43)$$

while Eq.(42) combined with Eq.(22) yields

$$\delta m_{12}^\ell \approx \cos \phi \sqrt{m_e m_\tau}, \quad \delta m_{13}^\ell \approx \sin \phi \sqrt{m_e m_\mu}, \quad \delta U_\mu \approx -\sin \phi \sqrt{m_e/m_\mu}, \quad \delta U_\tau \approx \cos \phi \sqrt{m_e/m_\tau}, \quad (44)$$

and

$$\delta m_{12}^\ell \approx \varepsilon_\ell \sqrt{m_e m_\mu}, \quad \delta m_{13}^\ell \approx \sqrt{m_e m_\mu}, \quad \delta U_\mu \approx -\sqrt{m_e/m_\mu}, \quad \delta U_\tau \approx \varepsilon_\ell \sqrt{m_e m_\mu}/m_\tau, \quad (45)$$

where $\tan^2 \phi \ll m_\tau/m_\mu$.

IV. NEUTRINO OSCILLATIONS

The neutrino oscillation parameters of Δm_{atm}^2 and Δm_\odot^2 are expressed in terms of our neutrino masses as

$$\Delta m_{atm}^2 \equiv m_{\nu 2}^2 - m_{\nu 3}^2 \approx m_\nu^2 \approx M_{12}^2 + M_{13}^2 \approx \delta m_{12}^\nu{}^2 + \delta m_{13}^\nu{}^2, \quad (46)$$

$$\Delta m_\odot^2 \equiv m_{\nu 1}^2 - m_{\nu 2}^2 \approx 4m_\nu m_{\nu 1}^{(1)} = 2m_\nu \left(\delta m_{11}^{rad} + \delta m_{22}^{rad} + \delta m_{33}^{rad} - m_{\nu 3}^{(1)} \right), \quad (47)$$

leading to

$$\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} = \left(\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \right)_{rad} + \left(\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \right)_{rot} \quad (48)$$

with

$$\left(\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \right)_{rad} = \frac{2(\delta m_{11}^{rad} + \delta m_{22}^{rad})}{m_\nu}, \quad \left(\frac{\Delta m_\odot^2}{\Delta m_{atm}^2} \right)_{rot} = 4 \frac{(M_{13}\delta U_\mu - M_{12}\delta U_\tau)(M_{12}N_{13} - M_{13}N_{12})}{m_\nu^3}. \quad (49)$$

The ratio $(\Delta m_\odot^2/\Delta m_{atm}^2)_{rad}$ arises from the radiative effects and $(\Delta m_\odot^2/\Delta m_{atm}^2)_{rot}$ receives the first order contributions from the rotation due to the diagonalization of $M_{\nu,\ell}$.

We are now in a position to estimating $\Delta m_\odot^2/\Delta m_{atm}^2$ to meet the LMA solution. Before going to its explicit evaluation, we list our assumptions on numerical values of the relevant mass parameters:

- $v_\chi \gg v_{weak} (= (2\sqrt{2}G_F)^{-1/2}=174 \text{ GeV})$ for $SU(3)_L \times U(1)_N \rightarrow SU(2)_L \times U(1)_Y$,
- $v_\eta = v_\rho = v_\ell^{(0)} = v_\ell^{(+)} = v_{weak}/2$ as the simplest case to keep $v_\eta^2 + v_\rho^2 + v_\ell^{(0)2} + v_\ell^{(+2)} = v_{weak}^2$ for weak boson masses,
- $m_{\eta^\pm} = m_{\rho^\pm} = m_{s^{(0)\pm}} = m_{s^{(+)\pm}} = v_{weak}$ and $m_{\chi^\pm} = |\mu_0| = v_\chi$,

where v_χ , presumably of order TeV, is left unspecified and is to be measured in the unit of $10v_{weak}$.

A. Atmospheric Neutrinos

Let us first reproduce $\Delta m_{atm}^2 \sim 3 \times 10^{-3} \text{ eV}^2$, which requires that $\delta m_{12}^{\nu^2} + \delta m_{13}^{\nu^2} \sim 3 \times 10^{-3} \text{ eV}^2$ from Eq.(46). The relation of $\delta m_{1i}^\nu = f_{1i} v_\nu^{(0)} = \delta m_{1i}^\ell v_\nu^{(0)} / v_\ell^{(0)}$ ($i=2,3$) together with Eqs.(43-45) for δm_{1i}^ℓ , then, leads to

$$v_\nu^{(0)} \approx (5.5, 1.8 / \cos \phi, 7.5) \times 10^{-9} v_\ell^{(0)}, \quad (50)$$

where three terms to be placed in the parenthesis, respectively, denote the cases with (hierarchical masses with $\delta m_{12}^\nu \approx \delta m_{13}^\nu$, democratic masses with $\delta m_{12}^\nu \gg \delta m_{13}^\nu$, democratic masses with $\delta m_{12}^\nu \ll \delta m_{13}^\nu$). In the following, the same notation is used. Since $v_\nu^{(0)}$ is expressed in terms of λ_2 and VEV's as estimated in Eq.(16), we find the constraint on λ_2 given by

$$-\lambda_2 \approx (1.1, 0.36 / \cos \phi, 1.5) v_\chi / 10 v_{weak} \times 10^{-7}. \quad (51)$$

Therefore, the magnitude of λ_2 is not arbitrary but fixed to be $|\lambda_2| = \mathcal{O}(10^{-7})$ by VEV's in order to obtain $\Delta m_{atm}^2 \sim 3 \times 10^{-3} \text{ eV}^2$.

B. Solar Neutrinos

Next, we calculate Δm_\odot^2 , which depends on the sum of δm_{11}^{rad} and δm_{22}^{rad} given by Eqs.(23) and (24) with $i = j = 2$. To find numerical values of $\delta m_{11,ij}^{rad}$, we use the following property of the integral I of Eq.(25):

$$I(0, a, a, b) \approx -1/ab (= -I_1), \quad I(0, a, a, a) \approx -1/a^2 (= -I_2). \quad (52)$$

By using the assumptions on mass parameters stated above, we know that the second term in Eq.(23) is proportional to I_1 and all other terms including δm_{ij}^{rad} of Eq.(24) are proportional to I_2 . In Eq.(23), the first term thus dominates the second term because of $I_2 \gg I_1$ for $m_{\chi^-}^2 \gg m_{s^{(0)+}}^2$ and $v_\chi \gg v_\rho$. The other terms proportional to I_2 , the third and fourth terms together with δm_{22}^{rad} , are more suppressed by the factor of $\lambda_{1,b} v_{weak}^2 / v_\chi^2$ compared with the first term since $|\lambda_{1,b}|$ are at most of order unity. Therefore, the first term in δm_{11}^{rad} gives the most dominant contributions to Δm_\odot^2 and those from all other terms can be safely neglected. The radiative mass of δm_{11}^{rad} saturated by the mediating τ lepton contribution turns out to be:

$$\delta m_{11}^{rad} \approx -73 \lambda_2 (1, \cos^2 \phi m_\tau / 2 m_\mu, 1/2) (v_\chi / 10 v_{weak})^3 \text{ eV}, \quad (53)$$

where, in the democratic cases, the factor of $m_\tau \cos^2 \phi / m_\mu$ is due to $\delta m_{12}^{\ell^2} (\propto f_{12}^2) = m_e m_\tau \cos^2 \phi$ in Eq.(44) and the factor $1/2$ arises from $c_\alpha^2 = s_\alpha^2 = 1/2$. The rotated couplings of F_{1i} used in these calculations are approximated to be $(f_{1i}, f_{12}/\sqrt{2}, \Theta f_{13}/\sqrt{2})$ ($i = 2, 3$) for $\Theta = -1$ for $i=2$ and $\Theta = 1$ for $i=3$.

By inserting the above result of Eq.(53) into $(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rad}$ shown in Eq.(49) with $m_\nu \sim 5.5 \times 10^{-2} \text{ eV}$, we reach the following estimate:

$$(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rad} \approx -2.7 \lambda_2 (1, 8.5 \cos^2 \phi, 0.5) (v_\chi / 10 v_{weak})^3 \times 10^3, \quad (54)$$

where $m_\tau / m_\mu \approx 17$ is used in Eq.(53). This estimate together with Eq.(51) for λ_2 yields

$$(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rad} \approx (2.9, 8.1 \cos \phi, 1.6) (v_\chi / 10 v_{weak})^4 \times 10^{-4}, \quad (55)$$

where $\cos \phi$ is bounded from below by $\tan^2 \phi \ll m_\tau / m_\mu$. By taking

$$v_\chi/10v_{weak} \approx \left(1.8 - 3.3, (1.4 - 2.5)^{4\sqrt{\cos\phi}}, 2.0 - 3.6\right), \quad (56)$$

we obtain $\Delta m_\odot^2 = (10^{-5} - 10^{-4}) \text{ eV}^2$ relevant to the LMA solution to the solar neutrino problem and

$$\delta m_{11}^{rad} = \Delta m_\odot^2 / 2m_\nu \approx 10^{-4} - 10^{-3} \text{ eV}. \quad (57)$$

Since $v_{weak}=174 \text{ GeV}$, v_χ roughly lies in the range of $3 - 6 \text{ TeV}$, which is the right order of $v_\chi (= \mathcal{O}(1 \text{ TeV}))$ as anticipated for the scale of $SU(3)_L \times U(1)_N \rightarrow SU(2)_L \times U(1)_Y$. For $v_\chi = 4 \text{ TeV}$ as a typical value, namely, $v_\chi/10v_{weak}=2.3$, we find that

$$(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rad} \approx (0.8, 2.3 \cos \phi, 0.6) \times 10^{-2}, \quad (58)$$

leading to $\Delta m_\odot^2 \sim (2.4, 6.9 \cos \phi, 1.8) \times 10^{-5} \text{ eV}^2$ for $\Delta m_{atm}^2 \sim 3 \times 10^{-3} \text{ eV}^2$.

C. Diagonalization Effect

Finally, we consider effects from $(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rot}$, which may disturb the behavior specific to the LMA solution indicated by $(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rad}$. Since $(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rot}$ is controlled by $m_{\nu 3}^{(1)}$ as in Eq.(49), it can be found that the leading terms of $\mathcal{O}(\delta U_{\mu,\tau})$ contained in $m_{\nu 3}^{(1)}$ turns out to vanish. The cancellation can be seen from the insertion of the leading terms of $M_{12,13} \approx N_{12,13}$ into the equation of $M_{12}N_{13} - M_{13}N_{12}$ appearing in the numerator of $m_{\nu 3}^{(1)}$ in Eq.(39) that turns out to vanish. The next nonvanishing contributions to $m_{\nu 3}^{(1)}$ are given by the second order terms in $M_{12}N_{13} - M_{13}N_{12}$ that result in the third order contributions to $m_{\nu 3}^{(1)}$, which is beyond our approximation because possible third order terms for δM_{ij} ($i, j=2,3$), which have not been included, directly give the third order contributions as can be seen from the definition of $m_{\nu 3}^{(1)}$ in Eq.(33). Therefore, we expect that $(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rot}$ becomes at most $\mathcal{O}(\delta U_\mu^3)$, which is about $(m_e/m_\mu)^{3/2} (= 3.4 \times 10^{-4})$ as shown in Eqs.(43)-(45), leading to $(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rot} \lesssim 10^{-4}$. This estimate of $(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rot}$ should be compared with Eq.(58) for the radiative case and in fact gives more suppressed contributions. It is, then, concluded that we can safely neglect $(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rot}$ and that the model can provide the LMA solution to the solar neutrino problem as indicated by $(\Delta m_\odot^2 / \Delta m_{atm}^2)_{rad}$.

Similarly, the third neutrino ν_3 will also acquire masses at most of order $m_\nu \delta U_\mu^3$, which is about 10^{-5} eV . The radiative correction to $m_{\nu 3}$ arising from δm_{33}^{rad} is estimated to be about

$$\delta m_{33}^{rad} \sim -0.2\lambda_2\lambda_b (1, m_\tau \cos^2 \phi / 2m_\mu, 0.5) (v_\chi/10v_{weak}) \text{ eV}. \quad (59)$$

Since $\lambda_2 \sim 10^{-7}$ and $\lambda_b \lesssim 1$, we find that $\delta m_{33}^{rad} \lesssim 0.5 \times 10^{-7} \text{ eV}$ for $v_\chi \sim 4 \text{ TeV}$. Thus, altogether we observe that the bound on $m_{\nu 3}$ is given by $m_{\nu 3} \lesssim 10^{-5} \text{ eV}$.

The cancellation of the diagonalization effects on the neutrino masses of $m_{\nu 1, \nu 3}^{(1)}$ is a reasonable consequence because the (1,2) and (1,3) entries of M_ν are exactly proportional to those of M_ℓ (see Eqs.(63)-(65)) so that the rotation effects due to $\delta U_{\mu,\tau}$, which induce $\delta M_{22} = 2m_\nu \cos \vartheta_\nu \delta U_\mu$, $\delta M_{23} = m_\nu (\cos \vartheta_\nu \delta U_\mu + \sin \vartheta_\nu \delta U_\tau)$, $\delta M_{33} = 2m_\nu \sin \vartheta_\nu \delta U_\tau$, are partially cancelled. However, the diagonalization effects manifest themselves in the matrix element U_{e3} that connects ν_e to ν_3 . The CHOOZ and PALOVERDE experiments imply that $|U_{e3}|^2 \lesssim 0.015 - 0.05$ [32]. For the mass matrix M_ν^{weak} of Eq.(29), this element is calculated to be

$$\begin{aligned} U_{e3} &= -\frac{1}{m_\nu} [(\cos^2 \vartheta_\nu - \sin^2 \vartheta_\nu) \delta M_{23} + \cos \vartheta_\nu \sin \vartheta_\nu (\delta M_{33} - \delta M_{22})] \\ &= -\frac{1}{m_\nu} [(c_\alpha \delta m_{13}^\nu + s_\alpha \delta m_{12}^\nu) \delta U_\mu - (c_\alpha \delta m_{12}^\nu - s_\alpha \delta m_{13}^\nu) \delta U_\tau], \end{aligned} \quad (60)$$

leading to $\delta U_\mu / \sqrt{2} \approx (1, -\sin \phi, -1) \sqrt{m_e / 2m_\mu}$ with $m_\mu / m_\tau \ll \tan^2 \phi \ll m_\tau / m_\mu$ for Eq.(44) allowing $|\delta U_\mu| \gg |\delta U_\tau|$, which includes the typical value of $\tan \phi \sim 1$. Our model predicts that $|U_{e3}|^2 \approx (1, \sin^2 \phi, 1) \times 0.0025$, which are consistent with the current experimental bound.

We have successfully demonstrated that the minimal 331 model with the global $L_e - L_\mu - L_\tau$ and Z_4 symmetry, which involves two (anti-)sextet Higgs scalars, $S^{(0,+)}$, indeed explains solar neutrino oscillations based on the LMA solution. The LMA solution is explicitly obtained by the use of the plausible parameter values for masses and couplings. The model has the following properties, which do not depend on the parameterization, that

1. the global $L_e - L_\mu - L_\tau$ symmetry is explicitly broken by $\eta\eta S^{(+c)}S^{(+c)}$ down to a discrete $Z_{L'}$ symmetry defined by $\exp(i\pi Q_{L'}/2)$ that forbids the L -breaking interactions for $S^{(+)}$, the absence of which is the essence of our discussions to be consistent with the bimaximal neutrino mixing scheme based on $L_e - L_\mu - L_\tau$;
2. the L conservation is explicitly broken by $\lambda_2\eta S^{(0)}\rho^c\chi^c$, which generates the VEV of $v_\nu^{(0)}$ for neutrinos calculated to be $\sim -\lambda_2 v_\eta v_\rho/v_\chi$, where $\lambda_2 < 0$ is the trigger for $v_\nu^{(0)} \neq 0$;
3. the mechanism generating the diagonal electron mass via δm_{1i}^ℓ ($i=2,3$) is linked to the requirement realizing the bimaximal neutrino mixing via δm_{1i}^ν owing to the proportionality of $\delta m_{1i}^\nu \propto \delta m_{1i}^\ell$;

and yields numerical results, which of course depend on the parameterization, that

1. the smallness of $m_{\nu 1} \approx m_{\nu 2} \sim 5.5 \times 10^{-2}$ eV for Δm_{atm}^2 is realized by $|\lambda_2|$ estimated to be $\mathcal{O}(10^{-7})$;
2. solar neutrino oscillations are induced by
 - the one-loop radiative effects, which yield

$$\Delta m_\odot^2/\Delta m_{atm}^2 \sim 10^{-2}, \quad (61)$$

corresponding to the LMA solution provided $SU(3)_L \times U(1)_N \rightarrow SU(2)_L \times U(1)_Y$ occurs at 3 – 6 TeV,

- the effects of the simultaneous diagonalization of the neutrino and charged-lepton mass matrices, which are expected to yield

$$\Delta m_\odot^2/\Delta m_{atm}^2 \lesssim (m_e/m_\mu)^{3/2} \sim 10^{-4}, \quad (62)$$

and which certainly yield the U_{e3} matrix element estimated to be $|U_{e3}| \sim \sqrt{m_e/2m_\mu} = 0.05$,

where, in the present model, the dominance of one-loop radiative effects yields the LMA solution. It should be stressed that the choice of $v_\chi = \mathcal{O}(1 \text{ TeV})$ automatically leads to the LMA solution.

The present radiative mechanism generates the Majorana mass of ν_e , δm_{11}^{rad} , given by the effective coupling of $(\eta^\dagger \psi^1) \psi^1 \chi (\chi S^{(+)})^c$. This coupling is induced by $\eta S^{(0)} \rho^c \chi^c$ and $\rho S^{(0)} \chi$ as in FIG.1 and is of the L' -conserving type. The less-dominant radiative masses are induced by $\psi^i \psi^j (S^{(+)})^c (S^{(0)})^c \eta \rho \chi$ for $\nu_e \nu_e$ with $i, j=1$ contributing to δm_{11}^{rad} and for $\nu_{\mu,\tau} \nu_{\mu,\tau}$ with $i, j=2,3$ contributing to δm_{ij}^{rad} . The $\nu_{\mu,\tau} \nu_{\mu,\tau}$ -terms involve the L' -breaking coupling of $\eta\eta S^{(+c)}S^{(+c)}$ as in FIG.2. The splitting of ν_1 and ν_2 is provided by δm_{11}^{rad} , which is about $10^{-4} - 10^{-3}$ eV, leading to the LMA solution. The mass of ν_3 is provided by δm_{33}^{rad} at most of order 10^{-7} eV but the diagonalization due to $M_{\nu,\ell}$ may give $m_{\nu 3} \lesssim 10^{-5}$ eV.

We have also presented plausible order-of-magnitude estimation of neutrino oscillations based on M_ν , which is correlated to M_ℓ :

1. for the hierarchical texture with $\delta m_{12}^\nu \sim \delta m_{13}^\nu$,

$$M_\nu = \epsilon_\nu \begin{pmatrix} * & \sqrt{m_e m_\mu} & \sqrt{m_e m_\mu} \\ \sqrt{m_e m_\mu} & * & * \\ \sqrt{m_e m_\mu} & * & * \end{pmatrix}, M_\ell = \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & \sqrt{m_e m_\mu} \\ \sqrt{m_e m_\mu} & \delta M & \Delta M \\ \sqrt{m_e m_\mu} & \Delta M & M \end{pmatrix}, \quad (63)$$

2. for the democratic texture with $\delta m_{12}^\nu \gg \delta m_{13}^\nu$,

$$M_\nu = \epsilon_\nu \begin{pmatrix} * & c_\phi \sqrt{m_e m_\tau} & s_\phi \sqrt{m_e m_\mu} \\ c_\phi \sqrt{m_e m_\tau} & * & * \\ s_\phi \sqrt{m_e m_\mu} & * & * \end{pmatrix}, M_\ell = \begin{pmatrix} 0 & c_\phi \sqrt{m_e m_\tau} & s_\phi \sqrt{m_e m_\mu} \\ c_\phi \sqrt{m_e m_\tau} & M & M - \Delta M \\ s_\phi \sqrt{m_e m_\mu} & M - \Delta M & M \end{pmatrix} \quad (64)$$

with $c_\phi = \cos \phi$ and $s_\phi = \sin \phi$ constrained by $\tan^2 \phi \ll m_\tau/m_\mu$,

3. for the democratic texture with $\delta m_{12}^\nu \ll \delta m_{13}^\nu$,

$$M_\nu = \epsilon_\nu \begin{pmatrix} * & \varepsilon_\ell \sqrt{m_e m_\mu} & \sqrt{m_e m_\mu} \\ \varepsilon_\ell \sqrt{m_e m_\mu} & * & * \\ \sqrt{m_e m_\mu} & * & * \end{pmatrix}, M_\ell = \begin{pmatrix} 0 & \varepsilon_\ell \sqrt{m_e m_\mu} & \sqrt{m_e m_\mu} \\ \varepsilon_\ell \sqrt{m_e m_\mu} & M & M - \Delta M \\ \sqrt{m_e m_\mu} & M - \Delta M & M \end{pmatrix} \quad (65)$$

with $\varepsilon_\ell \ll 1$,

where $\epsilon_\nu = v_\nu^{(0)}/v_\ell^{(0)}$, $M \gg \Delta M, \delta M$ and the asterisks stand for the suppressed entries to generate Δm_\odot^2 . The partial “degeneracy” between M_ν to M_ℓ suppresses the rotation effects on Δm_\odot^2 due to the diagonalization of M_ν and M_ℓ . As a result, the solar neutrino oscillations are, respectively, characterized by $\Delta m_\odot^2 \sim (2.4, 6.9 \cos \phi, 1.8) \times 10^{-5} \text{ eV}^2$ for $v_\chi \sim 4 \text{ TeV}$. Of course, if our mechanism explaining neutrino oscillations works further, one should next consider some reasons for the emergence of the approximate equality of $\delta m_{12}^\nu \sim \delta m_{13}^\nu$ in the hierarchical case and of the hierarchy of either $\delta m_{12}^\nu \gg \delta m_{13}^\nu$ or $\delta m_{12}^\nu \ll \delta m_{13}^\nu$ in the democratic case.

The masses of Higgs scalars are simply assumed to be about v_{weak} for those related to the weak boson masses for the order-of-magnitude estimate for radiative neutrino masses, namely, $m_{\eta^\pm} \sim m_{\rho^\pm} \sim m_{s(0)^\pm} \sim m_{s(+)\pm} \sim v_{weak}$. On the other hand, the χ scalar that generates masses for the exotic quarks and gauge bosons is assumed to have $m_{\chi^+} \sim v_\chi (\gg v_{weak})$ because $\langle 0|\chi|0 \rangle$ is related to the spontaneous breakdown of $SU(3)_L \times U(1)_N$ to $SU(2)_L \times U(1)_Y$. We have estimated v_χ to lie in 3 – 6 TeV in order to reproduce the observed neutrino mass pattern. It is further known that the 331 model contains peculiar particles, *i.e.* doubly-charged bilepton gauge bosons [33] with masses of the order $ev_\chi/\sqrt{2}$, that couple to $\bar{\ell}_L^c \gamma_\mu \ell_L$. Since v_χ can be 3 – 6 TeV, the bilepton gauge bosons can be as light as and even lighter than 1 TeV. It is, therefore, quite conceivable that we will observe the bilepton gauge bosons with masses around 1 TeV, which provide striking phenomena with clean $\ell^+ \ell^-$ jets [34].

Now equipped with the plausible mechanism that generates phenomenologically consistent masses for neutrinos and charged leptons, the 331 model may be a physically interested model of leptons. It not only explains three families of quarks and leptons because of the specific anomaly-cancellation mechanism used together with the asymptotic free condition on $SU(3)_c$ but also involves Majorana mass terms demanded by charged-lepton mass terms. There remain dynamical questions about the smallness of the L -violating coupling of λ_2 and the fine tuning of the lepton mass parameters to reproduce $m_e \ll m_\mu \ll m_t$, which require some new physics beyond $SU(3)_L \times U(1)_L$. It is also required to have extensive analyses on physics of the quark and gauge boson sectors as well to fully clarify potential power of the 331 model.

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Table Captions

TABLE I. L' , Z_4 and $Z_{L'}$ for leptons and Higgs scalars.

TABLE II. Z_4 and $Z_{L'}$ for non-self-Hermitian Higgs interactions.

Figure Captions

FIG. 1. One-loop diagrams for $\nu_e \nu_e$ with $\ell^i = \mu$ ($i=2$) and τ ($i=3$), where $m_{\ell^i} = F_{ii} v_\ell^{(+)}$ and F_{1i} is the rotated f_{1i} -coupling defined by $F_{12} = \cos \alpha f_{12} - \sin \alpha f_{13}$ and $F_{13} = \sin \alpha f_{12} + \cos \alpha f_{13}$.

FIG. 2. The same as FIG.1 but for $\nu_{\mu,\tau} \nu_{\mu,\tau}$ with $\nu_L^{i,j} = \nu_{\mu L}$ ($i, j=2$) and $\nu_{\tau L}$ ($i, j=3$).

TABLE I. L' , \mathcal{L} , Z_4 and $Z_{L'}$ for leptons and Higgs scalars, where the lepton number L is related to \mathcal{L} as $L = 4\lambda^8/2\sqrt{3} + \mathcal{L}$.

	ψ^1	$\psi^{2,3}$	η	ρ	χ	$S^{(0)}$	$S^{(+)}$
L'	1	-1	0	0	0	0	2
\mathcal{L}	1/3	1/3	-2/3	-2/3	4/3	-2/3	-2/3
Z_4	+	-	+	i	i	-	+
$Z_{L'}$	i	$-i$	+	+	+	+	-

TABLE II. Z_4 and $Z_{L'}$ for non-self-Hermitian Higgs interactions.

interactions	Z_4	$Z_{L'}$	interactions	Z_4	$Z_{L'}$	interactions	Z_4	$Z_{L'}$	interactions	Z_4	$Z_{L'}$
$S^{(0)} S^{(0)} S^{(0)}$	-	+	$S^{(0)} S^{(0)} S^{(+)}$	+	-	$S^{(0)} S^{(+)} S^{(+)}$	-	+	$S^{(+)} S^{(+)} S^{(+)}$	+	-
$\eta \eta S^{(0)c} S^{(0)c}$	+	+	$\eta \eta S^{(0)c} S^{(+c)}$	-	-	$\eta \eta S^{(+c)} S^{(+c)}$	+	+	$\rho \chi S^{(0)c} S^{(0)c}$	-	+
$\rho \chi S^{(0)c} S^{(+c)}$	+	-	$\rho \chi S^{(+c)} S^{(+c)}$	-	+	$\eta S^{(0)} \rho^c \chi^c$	+	+	$\eta S^{(+)} \rho^c \chi^c$	-	-
$\rho S^{(0)} \rho^c \eta^c$	-	+	$\rho S^{(+)} \rho^c \eta^c$	+	-	$\chi S^{(0)} \chi^c \eta^c$	-	+	$\chi S^{(+)} \chi^c \eta^c$	+	-
$\eta S^{(0)} \eta$	-	+	$\eta S^{(+)} \eta$	+	-	$\rho S^{(0)} \chi$	+	+	$\rho S^{(+)} \chi$	-	-
$\eta \rho \chi$	-	+	$(\rho^\dagger \eta)(\chi^\dagger \eta)$	-	+	$\text{Tr}(S^{(0)\dagger} S^{(+)})$	-	-			

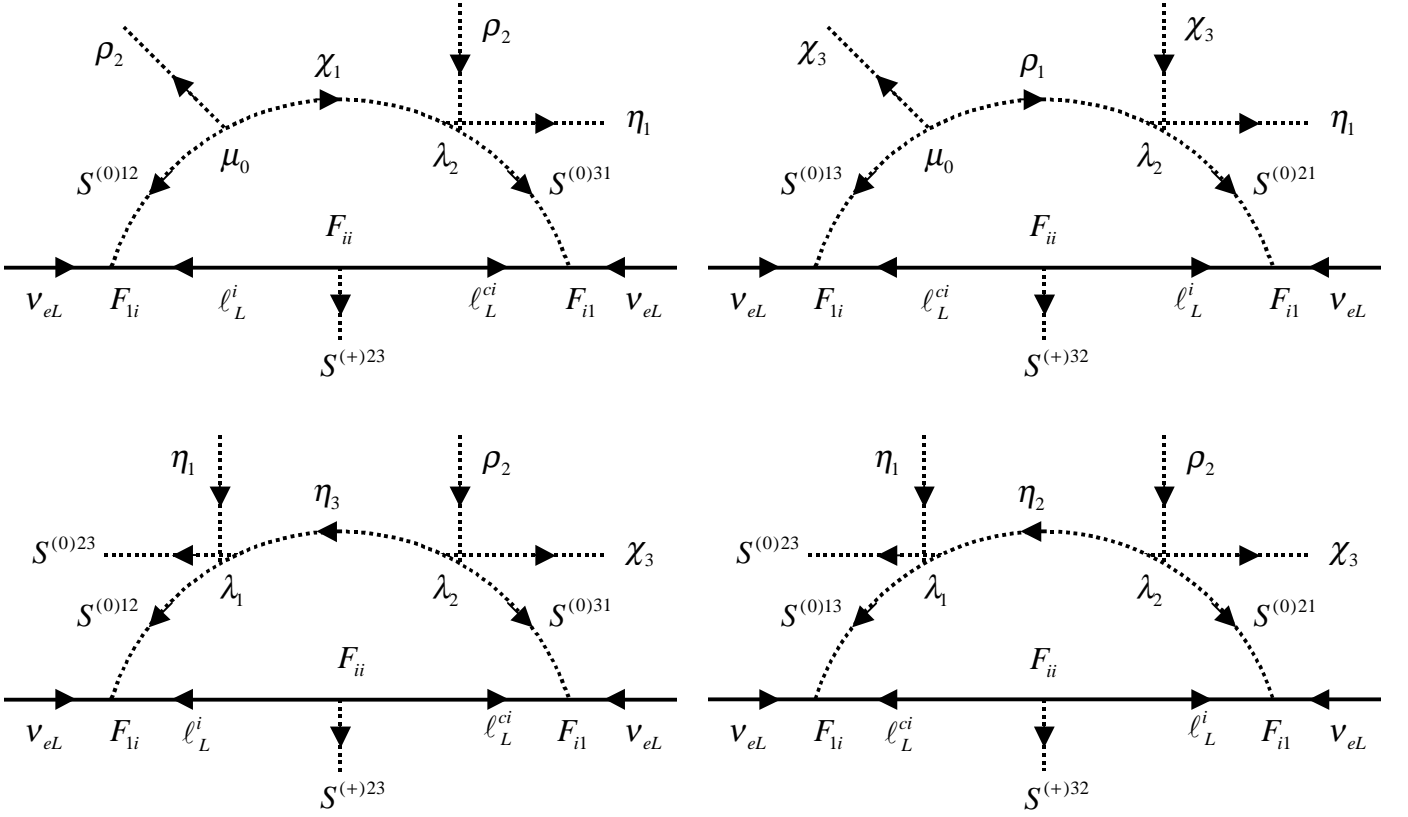


FIG.1. One-loop diagrams for $v_{eL}v_{eL}$ with $\ell^i = \mu$ ($i=2$) and τ ($i=3$), where $m_{\ell^i} = F_{ii} v_{\ell}^{(+)}$ and F_{li} is the rotated f_{li} -coupling defined by $F_{12} = \cos \alpha f_{12} - \sin \alpha f_{13}$ and $F_{13} = \sin \alpha f_{12} + \cos \alpha f_{13}$.

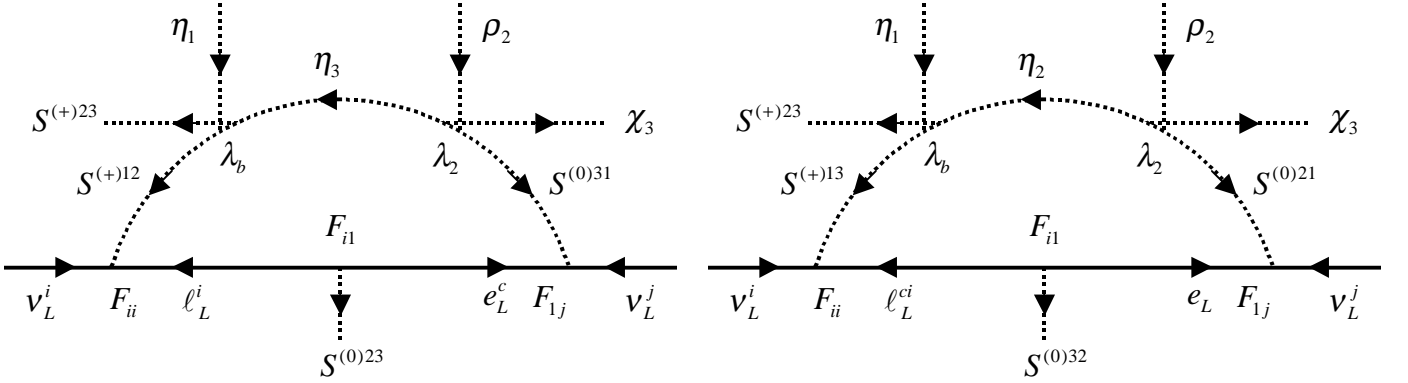


FIG.2. The same as FIG.1 but for $v_{\mu L, \tau L}$ with $v_L^{i,j} = v_{\mu L}$ ($i, j=2$) and $v_{\tau L}$ ($i, j=3$).